

Dissipative Interaction and Anomalous Surface Absorption of Bulk Phonons at a Two-Dimensional Defect in a Solid

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Abstract

We predict an extreme sensitivity to the dissipative losses of the resonant interaction of bulk phonons with a 2D defect in a solid. We show that the total resonant reflection of the transverse phonon at the 2D defect, described earlier without an account for dissipation, occurs only in the limit of extremely weak dissipation and is changed into almost total transmission by relatively weak bulk absorption. Anomalous surface absorption of the transverse phonon, when one half of the incident acoustic energy is absorbed at the 2D defect, is predicted for the case of “intermediate” bulk dissipation.

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Two-dimensional (planar) defects in the bulk of a solid, such as stacking faults, grain and twin boundaries, dislocation walls, substantially modify the vibrational spectrum of the system. Particularly strong effects can occur at resonant interaction of bulk phonons with a 2D defect in a solid, namely the total reflection of the grazing acoustic wave at an ultrathin (2D) embedded layer [1,2], the total transmission at resonance with intrinsic dynamical degrees of freedom of a 2D defect layer [3,4], the total reflection of a bulk phonon at resonance with an asymmetric vibrational mode of an *inhomogeneous* 2D elastic layer with complex internal structure [5], and the total reflection of the transverse acoustic wave at resonance with the quasilongitudinal leaky wave of a *homogeneous* 2D elastic layer [6,7]. To elucidate the possibilities of the experimental observation of these wave phenomena, caused by relatively thin elastic layer sandwiched in a solid, one has to study the effect on them of other relatively small interactions. In such a way one can check whether these phenomena can “survive” in a real experimental system.

In the present work we analyse the effect of bulk and surface dissipation on the interaction of a transverse elastic wave (acoustic phonon) with a homogeneous 2D elastic defect layer in a solid. The interest in this study is stipulated by the fact that three characteristic lengths simultaneously appear in the problem, namely the wavelength, the effective thickness of the 2D defect, and the effective “dissipative length” (or effective mean free path of the bulk or surface excitations which cause the dissipation). We show that the fine interplay between these three lengths determines the resonant interaction of bulk phonons with the planar defect. We emphasize that a relatively weak bulk and surface dissipation substantially effects the total reflection of a transverse bulk phonon at a 2D defect layer at resonance with the quasilongitudinal pseudosurface (leaky) wave at the layer, described in Refs. [6,7] without an account for the dissipation. We show that the (bulk and surface) dissipation causes finite transmission of bulk phonons through the 2D defect for any angle of incidence, including the resonant one. This result is well consistent with the statement, known from the theory of the Kapitza thermal resistance, that generally the dissipation enhances the phonon transmission through the interface between two media (see, e.g., Refs. [8,9]). We predict that at resonance with the leaky wave the transmission and reflection amplitudes are determined mainly by the dimensionless ratio of the longitudinal bulk dissipative length a_l and effective thickness a^* of the 2D defect which is enhanced by the large factor $(a^*k_x)^{-2} \gg 1$, where k_x is the tangential component of the incident wavevector. This characteristic ratio is proportional to the bulk viscosity and thermal conductivity of the solid and is inversely proportional to the square of the frequency. Almost total reflection of the transverse bulk phonon at the 2D defect occurs only in the limit of extremely weak dissipation when this ratio is much less than unity. This limit in most cases is hard to reach experimentally. When this ratio has the order of or is larger than unity, the total resonance reflection of the transverse bulk phonon is changed into almost total transmission (for all angles of incidence, including the resonant one). The origin of such strong effect of the dissipation on resonance interaction of bulk phonons with a 2D defect is the large penetration depth of the quasilongitudinal leaky wave and the strong enhancement of its amplitude at resonance with the incident transverse phonon (cf. Refs. [6,7]). It causes an extremely strong enhancement of the surface absorption of the incident wave. The enhancement of surface absorption of an acoustic phonon incident from liquid helium at resonance with the leaky Rayleigh wave at the liquid helium - solid interface has been described theoretically [10] and investigated experimentally (for a recent

review, see Ref. [11]). In the case of a 2D defect layer sandwiched in a solid, the effect of bulk dissipation on resonance reflection is strongly enhanced in comparison with the interface between liquid and solid due to the presence of an additional small and frequency-dependent parameter which governs the interaction between the incident transverse phonon and the quasilongitudinal leaky wave, namely the parameter $a^*k_x \ll 1$ which is the ratio between the effective thickness of the layer and the wavelength. We show that the coefficient of surface absorption, which follows from the reflection and transmission coefficients with an account for the bulk and surface dissipation, exactly coincides with the coefficient of surface absorption which can be obtained directly with the use of the dissipative function of a solid. This confirms the consistency of the proposed description of the effect of bulk and surface dissipation on resonant reflection and refraction of elastic waves in dissipative media, which in general is a rather tough problem (see, e.g., Ref. [12]). We describe also the conditions for anomalous surface absorption of the transverse phonon at the 2D defect when one half of the incident acoustic energy is absorbed at the ultrathin elastic layer. This effect, which can be interesting from the experimental point of view, is similar to anomalous surface absorption of the grazing shear elastic wave in a thin gap between two solids caused by the dissipative van der Waals interaction between the solids [13]. In both cases, the anomalous surface (interface) absorption of the bulk phonon is stipulated by resonant dissipative interaction with the leaky elastic wave.

At first, we consider the interaction of the bulk transverse wave with frequency ω with a 2D defect placed on the $z = 0$ plane in an isotropic elastic solid without an account for the bulk and surface dissipation. In an isotropic solid we can introduce scalar and vector potentials φ and $\vec{\psi}$, which satisfy the equations of motion $\ddot{\varphi} = c_l^2 \Delta \varphi$, $\ddot{\vec{\psi}} = c_t^2 \Delta \vec{\psi}$ and describe the longitudinal and transverse elastic displacements in the solid, $\vec{u} = \mathbf{grad} \varphi + \mathbf{curl} \vec{\psi}$, where c_l and c_t are, respectively, longitudinal and transverse sound velocities. In the case when the plane of incidence coincides with the XOZ plane, the vector potential can be taken as $\vec{\psi} = (0, \psi, 0)$. For the transverse wave incident from the $z < 0$ semispace and polarized in the plane of incidence, the wave fields in the medium 1 ($z < 0$) and medium 2 ($z > 0$) have the following form:

$$\psi^{(1)} = \left(e^{ik_t \cos \theta_t z} + r_t e^{-ik_t \cos \theta_t z} \right) e^{ik_t \sin \theta_t x - i\omega t}, \quad (1)$$

$$\varphi^{(1)} = r_l e^{-ik_l (\cos \theta_l z + \sin \theta_l x) - i\omega t}, \quad (2)$$

$$\psi^{(2)} = d_t e^{ik_t (\cos \theta_t z + \sin \theta_t x) - i\omega t}, \quad (3)$$

$$\varphi^{(2)} = d_l e^{ik_l (\cos \theta_l z + \sin \theta_l x) - i\omega t}, \quad (4)$$

where θ_t is the angle of incidence of the transverse wave, θ_l is the angle of reflection and transmission of the longitudinal wave, $r_{t,l}$ and $d_{t,l}$ are the reflection and transmission amplitudes for the transverse and longitudinal waves, respectively, $k_{t,l} = \omega/c_{t,l}$ are bulk wavenumbers.

For the macroscopic description of the interaction of an elastic wave with a planar defect with the effective thickness much less than the wavelength, one can make use of a set of effective boundary (matching) conditions for the elastic displacements and stresses at the plane of the 2D defect (see, e.g., Refs. [1,2,5,14]). In the case of a homogeneous 2D defect which does not possess complex internal structure and intrinsic dynamical degrees of freedom, these boundary conditions can be reduced in the simplest case to the continuity of the elastic displacements u_i and to the discontinuity of surface-projected bulk stresses σ_{zi} ,

$i = 1, 2, 3$ (for more general boundary conditions which account for the discontinuity both of surface-projected bulk stresses and elastic displacements, see Refs. [5,15]). For the problem under consideration, the following boundary conditions will be used:

$$u_x^{(1)} - u_x^{(2)} = 0, \quad u_z^{(1)} - u_z^{(2)} = 0, \quad (5)$$

$$\sigma_{zz}^{(1)} - \sigma_{zz}^{(2)} = g_1 \frac{\partial^2 u_z}{\partial x^2} - \varrho_s \frac{\partial^2 u_z}{\partial t^2}, \quad (6)$$

$$\sigma_{zx}^{(1)} - \sigma_{zx}^{(2)} = \tilde{h}_{11} \frac{\partial^2 u_x}{\partial x^2} - \varrho_s \frac{\partial^2 u_x}{\partial t^2}, \quad (7)$$

where ϱ_s is the total (positive) mass of the defect layer per unit surface area, $g_1 = g_{xx}$ and $h_{11} = h_{xxxx}$ are, respectively, the components of the tensors of the surface (interface) stresses and elastic moduli, $\tilde{h}_{11} = h_{11} + g_1$.

For the components of the elastic displacements and stresses in an isotropic solid which enter the boundary conditions (5)-(7), one has the following expressions in terms of scalar and vector potentials:

$$u_x = \frac{\partial \varphi}{\partial x} - \frac{\partial \psi}{\partial z}, \quad u_z = \frac{\partial \varphi}{\partial z} + \frac{\partial \psi}{\partial x}, \quad (8)$$

$$\sigma_{zz} = 2\rho c_t^2 \left(\frac{\partial^2 \varphi}{\partial z^2} + \frac{\partial^2 \psi}{\partial x \partial z} \right) + \rho(c_l^2 - 2c_t^2) \left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} \right), \quad (9)$$

$$\sigma_{zx} = \rho c_t^2 \left(2 \frac{\partial^2 \varphi}{\partial x \partial z} + \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial z^2} \right), \quad (10)$$

where ρ is density of the solid.

Making use of Eqs. (1)-(10), we find the following expressions for the reflection and transmission amplitudes:

$$r_t = [2i\rho k_t \cos \theta_l (\varrho_{sz} \sin^2 \theta_t - \varrho_{sx} \cos^2 \theta_t) - \varrho_{sz} \varrho_{sx} k_l k_t \cos(\theta_l - \theta_t) \cos(\theta_l + \theta_t)] \frac{1}{\Delta}, \quad (11)$$

$$r_l = -[k_l k_t \varrho_{sx} \varrho_{sz} \cos(\theta_l - \theta_t) + i\rho(\varrho_{sz} k_l \cos \theta_l + \varrho_{sx} k_t \cos \theta_t)] \frac{2 \sin \theta_l \cos \theta_t}{\Delta}, \quad (12)$$

$$d_t = [2\rho \cos \theta_l - i\varrho_{sx} k_l \sin^2 \theta_l - i\varrho_{sz} k_l \cos^2 \theta_l] \frac{2\rho \cos \theta_t}{\Delta}, \quad (13)$$

$$d_l = [\varrho_{sz} k_l \cos \theta_l - \varrho_{sx} k_t \cos \theta_t] \frac{2i\rho \sin \theta_l \cos \theta_t}{\Delta}, \quad (14)$$

$$\Delta = [2\rho \cos \theta_t - i\varrho_{sz} k_l \cos(\theta_l - \theta_t)] [2\rho \cos \theta_l - ik_t \varrho_{sx} \cos(\theta_l - \theta_t)], \quad (15)$$

where the conservation law of the tangential component of the phonon momentum has been used ,

$$k_x = k_t \sin \theta_t = k_l \sin \theta_l, \quad (16)$$

and the following notations are introduced:

$$\varrho_{sx} = \varrho_s - \tilde{h}_{11} \frac{k_x^2}{\omega^2}, \quad (17)$$

$$\varrho_{sz} = \varrho_s - g_1 \frac{k_x^2}{\omega^2}. \quad (18)$$

From Eqs. (11)-(15) follows that in general one has $r_t \sim r_l \sim d_l \sim k_t a^* \ll 1$, $d_t \sim 1$, which corresponds to almost total transmission of the wave through the thin (2D) elastic layer with effective thickness $a^* \sim \varrho_{sx}/\rho$ much less than the wavelength λ [16]. But for the resonant (non-grazing) angle of incidence $\theta_t = \theta_t^{(r)}$, which corresponds to $\cos \theta_l^{(r)} \sim i k_x a^*$, the reflection amplitude substantially increases and reaches the value of unity, $r_t = 1$, while the transmission coefficient simultaneously turns into zero (cf. Refs. [6,7]). Indeed, for the “overcritical” angles of incidence $\theta_t > \arcsin(c_t/c_l)$, when the $\cos \theta_l$ is imaginary, one can introduce the inverse decay length κ for the exponential decay of the longitudinal wave field away from the planar defect: $\cos \theta_l = i \kappa c_l / \omega$. Then from the requirement of the total reflection $d_t = 0$, from Eq. (13) we find the following equation for the parameter κ :

$$2\rho\kappa - \varrho_{sx}k_x^2 + \varrho_{sz}\kappa^2 = 0. \quad (19)$$

This equation in general has two roots for the real parameter κ , which have the following form in the longwavelength limit $k_x \ll \rho/\sqrt{|\varrho_{sx}\varrho_{sz}|}$:

$$\kappa_1 = \frac{1}{2} \frac{\varrho_{sx}}{\rho} k_x^2 = \frac{\varrho_s}{2\rho} \left[1 - \frac{c_l^{(s)2}}{c_l^2} \right] k_x^2, \quad (20)$$

$$\kappa_2 = -2 \frac{\rho}{\varrho_{sz}} = -\frac{2\rho\omega^2}{\varrho_s\omega^2 - g_1 k_x^2}. \quad (21)$$

In the case of $\varrho_{sx} > 0$, the first root $\kappa_1 \ll k_x$ (20) corresponds to the pseudosurface (leaky) wave with $\omega^2 \approx c_l^2 k_x^2$ and almost longitudinal polarization, which can propagate along the planar defect with locally decreased longitudinal velocity $c_l^{(s)} = \sqrt{\tilde{h}_{11}/\varrho_s} < c_l$ (see Refs. [17,14,1,2,6,7]). If one neglects dissipation, resonant excitation of this quasilongitudinal leaky wave causes total reflection of the bulk transverse wave by the planar defect [6,7]. The second root $\kappa = \kappa_2$ (21) is unphysical one for homogeneous planar defect with finite surface mass ϱ_s . It is worth mentioning that Eq. (19) can be also obtained from the solution of the problem of the total “nontransmission” of the transverse acoustic wave through the planar defect for the overcritical angle of incidence θ_t , when the potentials have the following form:

$$\psi^{(1)} = (e^{iqz} + r_t e^{-iqz}) e^{ik_x x - i\omega t}, \quad \varphi^{(1)} = r_l e^{\kappa z + ik_x x - i\omega t}, \quad (22)$$

$$\psi^{(2)} = 0, \quad \varphi^{(2)} = d_l e^{-\kappa z + ik_x x - i\omega t}, \quad (23)$$

where $q = \sqrt{\omega^2/c_t^2 - k_x^2} = k_t \cos \theta_t$. One can show that the wave fields (22) and (23) satisfy boundary conditions (5)-(7), together with the relations (8)-(10), if the parameter κ is a solution of Eq. (19) and the reflection amplitude r_t in Eq. (22) has the unit modulus, $r_t = \exp(i\phi_r)$, which reflects the conservation law of the acoustic energy flux through the planar defect. With the use of the above notations for κ and q , from Eq. (11) we find the following expression for the reflection amplitude r_t under the condition (19):

$$r_t = \frac{\varrho_{sx}q + i\varrho_{sz}\kappa}{\varrho_{sx}q - i\varrho_{sz}\kappa}. \quad (24)$$

For the value of $\kappa = \kappa_1$ given by Eq. (20), from Eq. (24) follows that $r_t \approx 1$ (or $\phi_r \ll 1$) since $\kappa \ll (k_x, q)$.

In a similar way we can also investigate the total transmission of the transverse acoustic wave through the 2D defect. From the requirement $r_t = 0$, from Eq. (11) we obtain the following equation for the parameter κ ,

$$2\rho\kappa - \varrho_{sx}k_x^2 = \frac{\varrho_{sx}q^2\kappa}{\varrho_{sz}k_x^2}(2\rho + \varrho_{sz}\kappa), \quad (25)$$

which has two roots in the longwavelength limit $k_x \ll \rho/\sqrt{|\varrho_{sx}\varrho_{sz}|}$. In the case of $\varrho_{sx} \ll \varrho_{sz}$, which is realized for the planar defect with $1 - c_l^{(s)2}/c_l^2 \ll 1$, one of the roots of Eq. (25) is close to the first root $\kappa = \kappa_1$ (20) of Eq. (19). Another root of Eq. (25) is unphysical one for such a 2D defect. Equation (25) can be also obtained from the solution of the problem of the total “nonreflection” of the transverse acoustic wave at the planar defect for the overcritical angle of incidence, when the potentials have the following form:

$$\psi^{(1)} = e^{iqz + ik_x x - i\omega t}, \quad \varphi^{(1)} = r_l e^{\kappa z + ik_x x - i\omega t}, \quad (26)$$

$$\psi^{(2)} = d_t e^{iqz + ik_x x - i\omega t}, \quad \varphi^{(2)} = d_l e^{-\kappa z + ik_x x - i\omega t}. \quad (27)$$

One can show that the wave fields (26) and (27) satisfy boundary conditions (5)-(7) if the parameter κ is a solution of Eq. (25) and the transmission amplitude d_t in Eq. (27) has the unit modulus, $d_t = \exp(i\phi_d)$. From Eq. (13) we find the following expression for the transmission amplitude d_t under the condition (25):

$$d_t = \frac{2\rho\kappa - \varrho_{sx}k_x^2 + i\varrho_{sx}\kappa q}{2\rho\kappa - \varrho_{sx}k_x^2 - i\varrho_{sx}\kappa q}. \quad (28)$$

For the solution of Eq. (25) close to the value $\kappa = \kappa_1$ given by Eq. (20), from Eq. (28) follows that $d_t \approx 1$ (or $\phi_d \ll 1$).

Therefore at a 2D defect with $\varrho_{sx} \ll \varrho_{sz}$ (or $1 - c_l^{(s)2}/c_l^2 \ll 1$), the resonant conditions for the total reflection and total transmission of the transverse acoustic phonon through the planar defect are very close and one needs to account for the additional (relatively weak) interactions at the 2D defect to distinguish between these two phenomena. Dissipative interaction of bulk phonons with a 2D defect, caused by bulk and surface dissipation, is an example of such physically real interaction which is always present in a solid. (Even zero-point lattice oscillations at $T = 0$ are related, via the fluctuation-dissipation theorem, to the dissipative properties of the lattice).

Below we account for dissipation in two different ways. At first we calculate the coefficient of surface absorption R_s in the limit of relatively weak absorption, using the dissipative function of a solid and expressions (12) and (14) for the reflection and transmissions amplitudes r_l and d_l without an account for the dissipation. Then we calculate the reflection and transmission amplitudes r_t and d_t with an account for dissipation and obtain general expression for the coefficient of resonant surface absorption, making use of these amplitudes. The coefficient of surface absorption is determined as the ratio between the dissipated, E_d , and incident, E_{inc} , (time averaged) acoustic energies per unit time per unit surface area (see, e.g., Refs. [10,18,19]). The viscosity of the solid gives the following contribution to the energy dissipated in the bulk of a solid, $E_d^{(b)}$, and at a surface of a 2D defect, $E_d^{(s)}$:

$$E_d = \int_{-\infty}^{\infty} [\eta(\dot{u}_{ik} - \frac{1}{3}\dot{u}_{ll})^2 + \frac{1}{2}\zeta\dot{u}_{ll}^2]dz + \eta^{(s)}(\dot{u}_{\alpha\beta}^s - \frac{1}{2}\dot{u}_{\alpha\alpha}^s)^2 + \frac{1}{2}\zeta^{(s)}\dot{u}_{\alpha\alpha}^s{}^2 \equiv E_d^{(b)} + E_d^{(s)}, \quad (29)$$

where η and ζ are shear and second bulk viscosities, $\eta^{(s)}$ and $\zeta^{(s)}$ are shear and second surface viscosities, the upper index s denotes the values of the elastic displacements at the defect plane, the Greek indices take the numbers 1 and 2 and numerate the coordinate axis in the plane tangential to the 2D defect. The thermal conduction of the solid also gives the contribution to E_d (see, e.g., Ref. [18]) which for the brevity we will not consider explicitly. The acoustic energy, incident at the unit area of the planar defect per unit time by the transverse acoustic wave with amplitude u_{to} and equal to unity value of the vector potential ψ , has the following form (see Eqs. (1) and (8)):

$$E_{inc} = \frac{1}{2} \rho c_t \omega^2 \cos \theta_t |u_{to}|^2 = \frac{1}{2} \rho c_t^2 \omega q k_t^2. \quad (30)$$

Now we take into account that at resonance of total reflection at a 2D defect with $\varrho_{sx} > 0$, the quasilongitudinal leaky wave is excited (see Eqs. (19) and (20)). Since at resonance the transmission and reflection amplitudes for the longitudinal wave are large, $|r_l| \approx |d_l| \approx k_x/\kappa \gg 1$, see Eqs. (12) and (14), this quasilocalized wave gives the main contribution to surface losses for $\theta_t = \theta_t^{(r)}$. Then with the use of Eqs. (29) and (30) together with Eqs. (2), (4), (8), (12), (14), (17), and (20), we calculate the coefficient of surface absorption R_s in the limit of relatively weak absorption (when $R_s \ll 1$):

$$R_s = \frac{E_d}{E_{inc}} = \frac{4\rho}{\varrho_{sx}^2 c_l \tan \theta_t^{(r)}} \left[\frac{2\eta_{11}\rho}{\varrho_{sx} k_x^2} + \eta_{11}^{(s)} \right], \quad (31)$$

where $\eta_{11} = (4/3)\eta + \zeta$ and $\eta_{11}^{(s)} = \eta^{(s)} + \zeta^{(s)}$. [If one accounts for the contribution of thermal conduction in the solid to the bulk dissipative function $E_d^{(b)}$, see, e.g., Ref. [18], the same linear combination of the η_{11} and the coefficient of thermal conductivity, which determines the absorption coefficient for longitudinal bulk elastic wave, will appear in r.h.s. of Eq. (31) instead of η_{11} .] According to Eq. (31), the main contribution to surface losses is given by the bulk dissipation (viscosity and thermal conduction) in a solid, when $E_d^{(s)}/E_d^{(b)} = \eta_{11}^{(s)}/(\eta_{11}\delta) \ll 1$, due to large penetration depth $\delta = \kappa^{-1} = 2\rho/(\varrho_{sx} k_x^2) \sim \lambda^2/a^* \gg \lambda \gg a^*$ of the quasilongitudinal leaky wave. Therefore the coefficient of surface absorption R_s is inversely proportional to the square of the frequency in the limit of relatively weak absorption, and the account for the surface dissipation in R_s has a sense only in the case of anomalously strong surface absorption (when the surface dissipative length greatly exceeds the bulk one, $a_l^{(s)} \gg a_l$, see Eqs. (32) and (37) below).

Now we turn to the calculation of the reflection and transmission coefficients with an account for the dissipation which permits us to find the coefficient of surface absorption R_s not only in the limit of relatively weak absorption, when $R_s \ll 1$, but also in the case when it reaches the value of the order of unity. To account for the dissipation in the reflection/refraction of elastic waves, one can use the “dissipative acoustic theory” (see, e.g., Refs. [8,9,20]) in which the elastic moduli (and corresponding sound velocities and wavevectors) are assumed to be complex quantities in bulk equations of motion and boundary conditions for them. Since in the problem under consideration the main contribution to resonant surface absorption is given by the quasilongitudinal (leaky) wave, below we consider the longitudinal, bulk and surface, sound velocities as

$$c_l = c_{lo} - i\omega a_l, \quad c_l^{(s)} = c_{lo}^{(s)} - i\omega a_l^{(s)}. \quad (32)$$

In Eq. (32) and in the following, index o refers to the limit in which one neglects the dissipation. The parameter a_l in Eq. (32) is a bulk longitudinal “dissipative length” which is directly related to the absorption coefficient γ_l of the longitudinal bulk acoustic wave: $a_l = \gamma_l c_{lo}^2 / \omega^2$ (and $a_l = \eta_{11} / 2\rho c_{lo}$ if one does not account explicitly for the thermal conduction in the solid, cf. Eqs. (29), (31) and Ref. [18]). The damping of the transverse elastic waves can be also taken into account by introducing, similarly to Eq. (32), the transverse dissipative length a_t . For the low frequencies $\omega\tau \ll 1$ (where τ is an effective relaxation time of the excitations which cause the elastic wave absorption), one has $\gamma_{l,t} \propto \omega^2$ and the lengths $a_{l,t}$ do not depend on the frequency and are proportional to the effective mean free path l of the excitations. For the high frequencies $\omega\tau \gg 1$, in single crystals one has $\gamma_{l,t} \propto \omega$ and the lengths $a_{l,t}$ are inversely proportional to the frequency: $a_{l,t} = p_{l,t} c_{lo,to} / \omega$, where the dimensionless parameters $p_{l,t}$ do not depend on the frequency and the effective mean free path l and describe phenomenologically the absorption of bulk phonons in the ballistic regime (cf. Refs. [8,9,11]). In both regimes, in the hydrodynamic $\omega\tau \ll 1$ and ballistic $\omega\tau \gg 1$ ones, relatively weak sound absorption in a solid corresponds to $\omega a_{l,t} \ll c_{lo,to}$ (or $p_{l,t} \ll 1$). The dissipative lengths $a_{l,t}$ can vary in rather broad interval of values and are functions of the state of the solid (its temperature, impurity concentration, etc.). These lengths strongly depend on the type of the solid (metal, insulator, or semiconductor), and usually $a_l > a_t$. In crystal quartz these lengths are rather short, for instance $a_l \sim 6\text{\AA}$ both at room temperature [21] and at $T = 140\text{K}$ [22]. In metals whose Fermi surface can be approximated by the free-electron spherical Fermi surface (such as copper, sodium, lead, tin, and indium), the electron viscosity η_e is proportional to the electrical conductivity and one has $\eta_e \sim p_F N_e l_e$, where p_F , l_e and N_e are the Fermi boundary momentum, electron mean free path and the number of conduction electrons per unit volume, respectively. In this case for the low frequencies one has $a_l \sim (p_F / M_i c_l) l_e$, where M_i is a mass of an ion in the metal. In particular, for the copper one has $a_l \sim 2 \cdot 10^{-3} l_e$, and therefore for the low frequencies the longitudinal dissipative length a_l can have the order of a micron in pure enough copper in the limit of $T = 0$ (when l_e reaches 10^{-2} cm , see, e.g., Ref. [23]).

The parameter $a_l^{(s)}$ in Eq. (32) is the surface (longitudinal) dissipative length which is related to the surface viscosity $\eta_{11}^{(s)} = \eta^{(s)} + \zeta^{(s)}$ (see Eqs. (29) and (31)): $a_l^{(s)} = \eta_{11}^{(s)} / (2\rho_s c_{lo}^{(s)})$. The surface dissipative length enters the considered problem via the complex elastic modulus $\tilde{h}_{11} = \rho_s c_l^{(s)2}$ in the boundary condition (7). Interface roughness and near-surface lattice imperfections increase the damping of acoustic phonons in the vicinity of 2D defect. It means that effectively one can have $a_l^{(s)} \gg a_l$, which justifies in this case the account for the surface (interface) dissipation in addition to the bulk one. Such strong inequality between surface (interface) and bulk dissipative lengths can occur also for a 2D electron gas at an interface between two similar semiconductors (such as *GaAs/AlGaAs* heterojunction), since in (semi)conductors at low enough temperature these lengths for the low frequencies are proportional to the local electrical conductivity (see also Ref. [5]).

Even with an account for the dissipation in the reflection/refraction of elastic waves, we can still consider the k_x component of the incident wavevector as a real quantity. Then making use of Eq. (32) and assuming that $a_l^{(s)} \gg a_l$, we obtain the following expansions:

$$\kappa = \sqrt{k_x^2 - \frac{\omega^2}{c_l^2}} = \kappa_o - i a_l \frac{\omega^3}{c_{lo}^3 \kappa_o}, \quad (33)$$

$$\varrho_{sx} = \varrho_s \left[1 - \frac{c_l^{(s)2}}{c_t^2} \right] = \varrho_{sxo} + 2ia_l^{(s)} \frac{\omega \varrho_s c_{lo}^{(s)}}{c_{lo}^2}, \quad (34)$$

which are valid for $(a_l, a_l^{(s)})k_x \ll 1$ and $a_l k_x^3 \ll \kappa_o^2$ (or $a_l \ll \varrho_{sxo}^2 k_x / \rho^2 \sim a^{*2} k_x$). [At resonance with the quasilongitudinal leaky wave, the account for the imaginary part of the q component of the incident wavevector gives only relatively weak corrections, of order $a_t k_x \ll 1$, to the final expressions, see Eqs. (35)-(37) below]. Making use of Eqs. (33), (34), from Eqs. (11) and (13) we find the reflection and transmission coefficients $r_t^{(r)}$ and $d_t^{(r)}$ for the transverse bulk phonon at resonance with the leaky wave at a 2D defect with an account for the dissipation:

$$r_t^{(r)} = \frac{\varrho_{sxo}^3 c_{lo} \tan \theta_t^{(r)} k_x^2}{4\rho(2a_l \rho^2 c_{lo} + a_l^{(s)} \varrho_{sxo} c_{lo}^{(s)} \varrho_s k_x^2) + \varrho_{sxo}^3 c_{lo} \tan \theta_t^{(r)} k_x^2}, \quad (35)$$

$$d_t^{(r)} = \frac{4\rho(2a_l \rho^2 c_{lo} + a_l^{(s)} \varrho_{sxo} c_{lo}^{(s)} \varrho_s k_x^2)}{4\rho(2a_l \rho^2 c_{lo} + a_l^{(s)} \varrho_{sxo} c_{lo}^{(s)} \varrho_s k_x^2) + \varrho_{sxo}^3 c_{lo} \tan \theta_t^{(r)} k_x^2}. \quad (36)$$

[Using expansions (33), (34) and the corresponding expansion for the q wavevector component, by equating to zero each of the two brackets in the denominator Δ (15) of all reflection/refraction amplitudes one can also find with an account for dissipation the dispersion relations for two “sagittal” waves, with quasilongitudinal and quasitransverse polarizations, which can propagate along the 2D defect, cf. Refs. [1,2]]. With the help of Eqs. (35) and (36) one can readily obtain the coefficient of surface absorption R_s as a dimensionless difference between the incident and reflected/transmitted fluxes of acoustic energy:

$$R_s = 1 - |r_t|^2 - |d_t|^2 = \frac{8(2a_l \rho^2 c_{lo} + a_l^{(s)} \varrho_{sxo} c_{lo}^{(s)} \varrho_s k_x^2) \rho \varrho_{sxo}^3 c_{lo} \tan \theta_t^{(r)} k_x^2}{[4\rho(2a_l \rho^2 c_{lo} + a_l^{(s)} \varrho_{sxo} c_{lo}^{(s)} \varrho_s k_x^2) + \varrho_{sxo}^3 c_{lo} \tan \theta_t^{(r)} k_x^2]^2}. \quad (37)$$

Expressions (35)-(37) represent the main result of this work. The results of Refs. [6,7], namely $r_t^{(r)} = 1$, $d_t^{(r)} = 0$, follow from Eqs. (35), (36) in the limit in which one neglects the dissipation: $a_l = a_l^{(s)} = 0$. If the introduced relations between the dissipative lengths and viscosities are used in Eq. (37), namely $2a_l \rho c_{lo} = \eta_{11}$ and $2a_l^{(s)} \varrho_s c_{lo}^{(s)} = \eta_{11}^{(s)}$, the expression (37) for R_s coincides exactly with the expression (31) in the limit of relatively weak absorption. From Eqs. (31), (35)-(37) follows that this limit corresponds to extremely weak dissipation: $a_l \ll \varrho_{sx}^3 \tan \theta_t^{(r)} k_x^2 / 8\rho^3 \sim a^{*3} k_x^2 \ll a^*$ and $a_l^{(s)} \ll \varrho_{sx}^2 / \varrho_s \rho$. Only in this limit one has $|d_t^{(r)}| \ll 1$, $r_t^{(r)} \approx 1$, and $R_s \ll 1$. But in view of the abovementioned characteristic values of the bulk dissipative length a_l , especially in metals in which a_l can have the order of a micron, this limit is hard to reach experimentally because of the assumed smallness of the dimensionless parameter $a^* k_x$: one needs $\sqrt{a_l / a^*} \ll a^* k_x \ll 1$. [Moreover, the effective thickness of the layer $a^* = \varrho_{sx} / \rho = h(1 - c_l^{(s)2} / c_t^2)$, which enters Eqs. (35)-(37), is much smaller than the “actual” thickness $h = \varrho_s / \rho$ of the layer if $1 - c_l^{(s)2} / c_t^2 \ll 1$]. Therefore from the experimental point of view most plausible is the opposite limit $a_l \gg \varrho_{sx}^3 \tan \theta_t^{(r)} k_x^2 / 8\rho^3 \sim a^{*3} k_x^2$ (which is consistent with the initial assumptions of the proposed approach that $a_l \ll \varrho_{sx}^2 k_x / \rho^2 \sim a^{*2} k_x$ and $a^* k_x \ll 1$, see Eq. (33)). In this low-frequency limit bulk phonons traverse the 2D defect almost without reflection and surface absorption, when $r_t \ll 1$, $d_t \approx 1$, $R_s \ll 1$ for all

angles of incidence, including the resonant one (as it should be from the “intuitive” point of view). With the use of Eqs. (11), (13), (33), and (34) one can also ascertain that the dissipation does not considerably change the reflection and transmission coefficients for the transverse elastic wave at resonance of *total transmission*, described by Eq. (25), in contrast to the reflection/transmission coefficients at resonance of *total reflection*, described by Eq. (19). The total reflection of the grazing acoustic wave at a 2D defect layer [1,2] and the total reflection of a bulk phonon at resonance with an asymmetric vibrational mode of an “inhomogeneous” 2D elastic layer with complex internal structure [5] are also not affected strongly by the bulk dissipation since these phenomena are not accompanied by the resonant excitation of the deeply penetrating leaky wave. It means that resonant interaction with the deeply penetrating leaky wave and total reflection of the long transverse elastic wave at a “homogeneous” planar defect, described in Refs. [6,7] without an account for the dissipation, turns to be extremely sensitive to the dissipative losses in a solid, especially to the bulk ones. From the experimental point of view, an interest can represent an “intermediate” case when $a_l \sim \varrho_{sx}^3 \tan \theta_t^{(r)} k_x^2 / 8\rho^3 \sim a^{*3} k_x^2$. Indeed, from Eq. (37) follows that if $a_l = (\varrho_{sx}^3 c_l \tan \theta_t^{(r)} - 4\varrho_s \varrho_{sx} \rho a_l^{(s)} c_l^{(s)}) k_x^2 / 8\rho^3 c_l > 0$, the coefficient of surface absorption R_s reaches its maximal value 0.5 (when $r_t^{(r)} = d_t^{(r)} = 0.5$). It means that one half of the energy flux of the incident long acoustic wave is absorbed by a thin, with $a^* \ll \lambda$, elastic layer (2D defect) sandwiched in a solid. This effect of anomalous surface absorption is similar to anomalous absorption, also with $R_s = 0.5$, of the grazing shear elastic wave in a thin gap between two solids caused by the dissipative van der Waals interaction between the solids [13]. In both cases, the anomalous surface (interface) absorption of the bulk phonon is stipulated by resonant dissipative interaction with the leaky elastic wave.

In conclusion, we predict an extreme sensitivity to the dissipative losses, especially to the bulk ones, of the resonant interaction of bulk phonons with a thin elastic layer (2D defect) sandwiched in a solid. The coefficients of the resonant transmission and reflection of the transverse phonon at the 2D defect are derived with an account for the bulk and surface dissipation. We show that almost total reflection of the transverse phonon at a 2D defect, described in Refs. [6,7] without an account for the dissipation, occurs only in the limit of extremely weak dissipation when $a_l \ll a^{*3} k_x^2$ and $a_l^{(s)} \ll \varrho_{sx}^2 / \varrho_s \rho$, which is hard to realize experimentally. In this limit the coefficient of surface absorption R_s of the incident phonon is small and is proportional to the bulk viscosity and thermal conductivity of the solid, but is inversely proportional to the square of the frequency. We show that almost total resonant reflection of the transverse phonon at the 2D defect can be changed into almost total transmission by relatively weak bulk absorption. Anomalous surface absorption of the transverse phonon, when R_s reaches its maximal value 0.5 and which is caused by resonant dissipative interaction with the leaky elastic wave, is predicted for the case of intermediate bulk dissipation when $a_l \sim a^{*3} k_x^2$.

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